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AN IMPLICIT/EXPLICIT APPROACH TO MULTIOBJECTIVE OPTIMIZATION WITH AN APPLICATION TO FOREST MANAGEMENT PLANNING

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19. Abstract

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AN IMPLICIT/EXPLICIT APPROACH TO MULTIOBJECTIVE OPTIMIZATION WITH AN APPLICATION TO FOREST MANAGEMENT PLANNING

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KEYWORDS: Utility/value theory, multiobjective programming, forest management.

There are numerous ideas and techniques available for solving multiobjective optimization problems, and they have been surveyed, compared and classified many times (e.g., Chankong and Haimes[1983], Cohon[1978], Cohon and Marks[1975], Evans[1984], Goicoechea et al.[1982], Haimes et al.[1975], Harrison[1983], Ho[1979], Hwang and Masud[1979], Rosenthal[1985], Roy and Vinke[1981], and Zeleny[1982]). In this paper we identify two categories of multiobjective techniques - explicit utility/value maximization and implicit utility/value maximization - and we develop a hybrid technique which combines the strengths of both. We also report on the implementation of a forest management planning system which is based on this hybrid approach.

The multiobjective optimization problem is defined as follows. We are given a feasible region X and objective functions $f_1,\ldots,f_K:X\rightarrow R$. We must find

an $\mathbf{x} \in X$ which yields a most preferred value of $\mathbf{f}(\mathbf{x})=(f_1(\mathbf{x}),\ldots,f_K(\mathbf{x}))$ over the set $V=\{\mathbf{v}: \mathbf{v}=\mathbf{f}(\mathbf{x}) \text{ for some } \mathbf{x} \in X\}$.

Here one cannot avoid using a vague, subjective term like "most preferred" because the multiobjective optimization problem is not well-defined in any strict mathematical sense. Despite this drawback, the problem has received wide attention in theory and practice, and useful mathematical analyses have been brought to bear on it.

1. THE EXPLICIT AND IMPLICIT APPROACHES

The explicit utility/value maximization approach is to first specify a function $U:V \rightarrow R$ with the property that $U(\mathbf{v}_1) > U(\mathbf{v}_2)$ if and only if $\mathbf{v}_1$ is preferred to $\mathbf{v}_2$, and then solve

$$\max U(\mathbf{f}(\mathbf{x})) \text{ s.t. } \mathbf{x} \in X. \tag{1}$$

The techniques for assessing an appropriate U come from the field of multiattribute utility/value theory (e.g., Dyer and Sarin[1979], Farquhar[1984], Fishburn[1983], Keeney[1977], Keeney and Raiffa[1976], and Kirkwood and Sarin[1980]). In this literature, utility theory and value theory are distinguished by the presence or absence, respectively, of uncertainty.

The great advantage of this approach is that it makes the vast body of theory, algorithms, software and experience that currently exist for single-objective optimization immediately available for solving multiobjective problems.

In spite of this great advantage, the approach of combining multiattribute utility/value assessment with mathematical programming has been used infrequently and perhaps with relatively little notice. This is not due

to any lack of acceptance of multiattribute assessment techniques. To the contrary, these techniques have been used widely, but their application has been limited almost exclusively to situations in which X is so small that the maximization of U is readily performed by total enumeration. (For typical examples, see Hannan, Smith, and Gilbert[1983], Hobbs[1979], Keeney[1979,1980] and others. For rare exceptions, see Golabi, Kirkwood, and Sicherman[1981], Gros[1975], Harrison and Rosenthal[1986], Keefer[1978], and Ringuest and Gulledge[1983].) We argue that this is a circumstantial, not theoretical, restriction (Harrison and Rosenthal[1984]). Nonetheless, it is fair to say that the disadvantage (perhaps overestimated at times) of the explicit approach is the information burden of having to specify a utility/value function over all of V in advance of the optimization process.

The <u>implicit utility/value maximization</u> approach, introduced by Geoffrion, Dyer and Feinberg[1972] and extended significantly by many others, removes this disadvantage. The implicit approach is also known as the <u>interactive</u> approach because it relies on information obtained from the decision maker during the solution process. Geoffrion et al.'s suggestion was to attempt maximization of U without requiring explicit knowledge of the form of U. This approach assumes that U exists and that it possesses desirable properties (such as differentiability and concavity) but the approach never calls for the evaluation of U. The algorithm is based on the Frank-Wolfe nonlinear programming method (though other primal methods could be used as well). When the algorithm requires information about the function being optimized (e.g., a gradient value or a step size), it is obtained through a computer/decision maker interaction. This dialogue can be structured so that the decision maker's only task is to answer questions of the form: "which do

you prefer, $\mathbf{v}_1$ or $\mathbf{v}_2$, or are you indifferent?" This information is sufficient to enable all other steps of the nonlinear programming algorithm to be executed by the computer (Dyer[1973]). Remarkably, under the assumption of concave utility (which is consistent with economic theory, e.g., Baumol[1977]), this procedure converges to optimality. It may not be possible to carry out the procedure long enough to achieve convergence, however.

Extensions and improvements to the implicit approach have been made by Oppenheimer[1978], and Zionts and Wallenius [1976,1983], among others. One of Zionts and Wallenius' most important improvements is to guarantee that each iteration yields an efficient solution. A point is efficient if it is impossible to find another point which is better with respect to some objective and no worse with respect to all other objectives. Other improvements involve ways of making the implicit approach converge faster. In spite of the improvements, the disadvantage of the implicit approach is that it cannot fully exploit the power of the mathematical programming algorithm upon which it is based. This is because human interaction is required at each iteration of the algorithm, so it is impossible to execute as many iterations of an implicit procedure as one routinely performs of a purely computational one.

Thus, the implicit and explicit approaches can be regarded as complementary in their strengths and weaknesses. The explicit approach can take full advantage of single-objective optimization technology but it has a large information burden in terms of the fully specified utility/value function. The implicit approach has a much smaller information burden at each iteration, requiring only paired comparisons or other local information, but it is much less effective at exploiting optimization technology.

The approach of this paper is a hybrid of the implicit and explicit approaches, extending and combining the strengths of both. Developing a hybrid seems obvious when the complementarity of the implicit and explicit approaches is exposed. However, except for the work of Oppenheimer[1978], this idea has been overlooked. We build and expand on Oppenheimer's work, demonstrate theoretical justification, and discuss the application of this idea to a problem in forest management that motivated the development of our implicit/explicit algorithm.

2. Structure of the Implicit/Explicit Approach

The key idea of the implicit approach is that, even though U is unknown, the direction of the gradient $\nabla_{\mathbf{x}} U(\mathbf{f}(\mathbf{x}))$ can be approximated (Geoffrion et al.[1972]). The approximation requires the selection of one objective, say f_1, called the <u>reference objective</u>, and the assessment of <u>marginal rates of substitution</u>

$$MRS_i(\mathbf{f}(\mathbf{x})) = \frac{\partial U(\mathbf{f}(\mathbf{x}))}{\partial f_i} \bigg/ \frac{\partial U(\mathbf{f}(\mathbf{x}))}{\partial f_1} . \tag{2}$$

If $\partial U/\partial f_1 \neq 0$, then

$$\nabla_{\mathbf{x}} U(\mathbf{f}(\mathbf{x})) = \frac{\partial U(\mathbf{f}(\mathbf{x}))}{\partial f_1} \sum_{i=1}^{K} MRS(\mathbf{f}(\mathbf{x}))\ \nabla f_i(\mathbf{x}) . \tag{3}$$

Geoffrion et al. assume $\partial U/\partial f_1 > 0$ at all points (i.e., the decision maker's desire for more f_1 is insatiable), in which case the first term on the right-hand side of (3) can be disregarded as a scaling factor and the vector

$$\sum_{i=1}^{K} MRS(\mathbf{f}(\mathbf{x}))\ \nabla f_i(\mathbf{x}) \tag{4}$$

can be used as a surrogate for $\nabla_{\mathbf{x}} U$ in the nonlinear programming algorithm used to maximize U.

Without explicit knowledge of U, we can not expect to assess MRS_i exactly, but Dyer[1973] has shown how to obtain approximate values through a series of paired comparisons between elements of V. The drawback of this approach, as noted, is that there needs to be interaction with the decision maker every time the gradient is to be evaluated, and therefore only a relatively small number of nonlinear programming iterations are possible.

In the implicit/explicit approach we attempt to lessen this dependence on the decision maker. However, we do not let the decision maker remain uninvolved in the value maximization process, as in the explicit approach. The idea of the implicit/explicit approach is as follows. First, we perform a few iterations of the implicit approach during which we accumulate information about the decision maker's preferences. Second, we use this information to approximate an explicit representation of the decision maker's value function. Third, we use this explicit function to define an explicit value maximization problem (1), which we solve to optimality. We can then repeat this cycle, until the change in the solution between successive cycles is negligible.

There are several important issues that we must resolve and specify clearly before the above idea can be implemented.

1. What form should be assumed for the explicit value function?
2. How should the parameters of the explicit value function be determined?

3. How can we solve problem (1) with sufficient speed so that the explicit optimization step can be embedded within an interactive approach?

4. What should be done if the approximating explicit value function is so inaccurate that the optimal solution to (1) is not preferred by the decision maker to the previous incumbent solution?

5. How do we accommodate inconsistencies on the part of the decision maker, especially early in the interactive process?

These are the major issues of implementation of the implicit/explicit approach, and we address them in the following sections. Some issues can be addressed with generality; others, particularly the third, are best approached on an application-specific basis. This third question is especially challenging in our forestry application, because problem (1) in this case is fairly complex and our software is implemented on a microcomputer.

3. Forms of the Explicit Value Function

Like Oppenheimer[1978], we have developed the implicit/explicit approach for two different forms of the explicit value function U. We use the deterministic additive and multiplicative forms found, e.g., in Dyer and Sarin[1979]. Both of these forms require, for each f_i, the determination of a *single-attribute value function* (SAVF), $u_i(f_i)$, which maps achievement of f_i onto the interval [0,1]. This function is monotone increasing, with $u^{-1}(1)$ the most desirable level of f_i, and $u^{-1}(0)$ the least desirable level. A common choice (e.g., Keeney[1979]) is an exponential SAVF:

$$u_i(f_i) = (1 - \exp(b_i))^{-1} (1 - \exp(b_i f_i)) \tag{5}$$

where $b_i<0$. We assume that f_i has been scaled to have range [0,1], and that the decision maker is insatiable with respect to f_i, i.e. $u^{-1}(0)=0$, $u^{-1}(1)=1$.

The additive form of $U(\mathbf{f})$ requires the determination of weighting parameters w_i such that

$$\sum_{i=1}^{K} w_i = 1, \quad w_i \geq 0,$$

and then

$$U(\mathbf{f}(\mathbf{x})) = \sum_{i=1}^{K} w_i \, u_i(f_i(\mathbf{x})). \tag{6}$$

The multiplicative form requires parameters $k_0, k_1, \ldots, k_K$ in addition to the SAVF's. It takes the form

$$U(\mathbf{f}(\mathbf{x})) = k_0^{-1} \{ \prod_{i=1}^{K} [1 + k_0 \, k_i \, u_i(f_i(\mathbf{x}))] - 1\} \tag{7}$$

subject to

$$1 + k_0 = \prod_{i=1}^{K} (1 + k_0 \, k_i) \tag{8}$$

$$0 < k_i < 1, \quad i=1,\ldots,K \tag{9}$$

$$k_0 > -1, \text{ and } k_0 \neq 0. \tag{10}$$

The underlying assumptions and axiomatic bases of these forms are given in the decision theory literature, e.g., by Dyer and Sarin[1979], Farquhar[1984], Fishburn[1983], Keeney and Raiffa[1976] and Kirkwood and Sarin[1980].

4. Parameter Estimation for the Explicit Value Problem

Depending on the choice of the functional form for explicit value, we must solve a constrained nonlinear least squares problem to determine the function's parameters. In both the additive and multiplicative cases, our approach to this problem is to derive an analytic expression for $MRS_i(\mathbf{f}(\mathbf{x}))$ as

a function of the unknown parameters and then solve for the minimum sum of squared deviations between the derived and observed values. We allow the user to specify the number of observed MRS values of $\mathbf{x}$ and MRS ($\geq$ K) to use in this estimation problem. In other words, the user determines the number of iterations of the implicit approach to do in the first step of the implicit/explicit cycle. This is a departure from Oppenheimer who uses a fixed number of implicit iterations every cycle. We derive analytic MRS expressions and formulate the resulting parameter estimation problems below for the additive and multiplicative cases.

4.1 <u>Formulation in the Additive Case</u>

In the additive case with exponential SAVFs, the marginal rate of substitution derived from (2), (5) and (6) is

$$MRS_i(\mathbf{f}(\mathbf{x})) = [w_1\ b_1\ (1-\exp(b_i)]^{-1}\ [w_i\ b_i\ (1-\exp(b_1)]$$
$$[\exp(b_i f_i(\mathbf{x}) - b_1 f_1(\mathbf{x}))]. \qquad (11)$$

Suppose the observed MRS values in the q'th implicit iteration are y_i^q, $i=1,\ldots,K$, $q=1,\ldots,m$ and the objective levels are f_i^q. Substituting the observations into (11) yields

$$MRS_i^q(\mathbf{w},\mathbf{b}) = [w_1\ b_1\ (1-\exp(b_i)]^{-1}\ [w_i\ b_i\ (1-\exp(b_1)]$$
$$[\exp(b_i f_i^q - b_1 f_1^q)]. \qquad (12)$$

The parameter estimation problem in the additive case is then the nonlinear program in $\mathbf{w}$ and $\mathbf{b}$ given by

$$\min \sum_{q=1}^{m} \sum_{i=1}^{K} (MRS_i^q(\mathbf{w},\mathbf{b}) - y_i^q)^2 \tag{13}$$

s.t.

$$\sum_{i=1}^{K} w_i = 1 \tag{14}$$

$$\epsilon \le w_i \le 1-\epsilon \quad i=1,\ldots,K \tag{15}$$

$$b_i \le -\epsilon \quad i=1,\ldots,K. \tag{16}$$

where ϵ is a small positive constant to insure that $b_i<0$ and w_i is strictly between 0 and 1. The reason for the redundant upper bound in (15) is that it turns out we can solve the nonlinear program while ignoring constraint (14). This convenience is guaranteed by the following result.

> ***Lemma 1*** (Harrison[1983] pp. 70-71): Let $(\mathbf{w},\mathbf{b})$ be optimal in the problem defined by (13), (15), (16). Then $(c\mathbf{w},\mathbf{b})$ is optimal in (13) - (16) where
>
> $$c = \left[\sum_i w_i \right]^{-1}.$$

Due to this result, we can regard the parameter estimation problem as a nonlinear program constrained only by simple bounds.

4.2 Formulation in the Multiplicative Case

For the case of a multiplicative value function we derive an equation from (2) and (7), analogous to (11), and substitute the q^{th} observation, f_i^q, y_i^q into it, analogous to (12). This yields

$$\mathrm{MRS}_i^q(\mathbf{k},\mathbf{b}) = \frac{(k_i\ b_i)\exp(b_i f_i^q)(1-\exp(b_1))\left[1 + k_0\ k_1 \frac{(1-\exp(b_1 f_1^q))}{(1-\exp(b_1))}\right]}{(k_1\ b_1)\exp(b_1 f_1^q)(1-\exp(b_i))\left[1 + k_0\ k_i \frac{(1-\exp(b_i f_i^q))}{(1-\exp(b_i))}\right]} \tag{17}$$

and the resulting formulation of the parameter estimation problem for the multiplicative case is the nonlinear program in $\mathbf{k}$ and $\mathbf{b}$ given by

$$\min \sum_{q=1}^{m} \sum_{i=1}^{K} (\mathrm{MRS}_i^q(\mathbf{k},\mathbf{b}) - y_i^q)^2 \tag{18}$$

$$\text{s.t.} \quad 1 + k_0 = \prod_i (1 + k_0\ k_i) \tag{19}$$

$$b_i \le -\epsilon \qquad i=1,\ldots,K \tag{20}$$

$$\epsilon \le k_i \le 1-\epsilon, \quad i=1,\ldots,K \tag{21}$$

$$k_0 > -1+\epsilon, \tag{22}$$

$$k_0 \ne 0. \tag{23}$$

This problem turns out to be much more amenable to solution than one would suspect considering the forboding appearance of (17), and the nonlinear constraint (19). Again, a key result is the freedom to ignore the non-bound constraint (19).

> ***Lemma 2.*** (Harrison[1983], pp. 81-83.): Let $(\mathbf{k},\mathbf{b})$ be optimal in the problem defined by (18), (20) - (23). Then $(\mathbf{k}',\mathbf{b})$ is optimal in (18) - (23) where
>
> $$k_0' = k_0/c$$
>
> $$k_i' = c\ k_i,\ i=1,\ldots,K, \text{ and}$$
>
> $$c = k_0\ [\ \prod_i (\ 1 + k_0\ k_i) - 1]^{-1}.$$

As for the nonconvex constraint (23), one obvious approach would be to separate the problem into two cases, $k_0 > 0$ and $k_0 < 0$. This turns out to be

unnecessary, however. The original reason for requiring $k_0 \neq 0$ is that k_0 appears as a quotient in (7). But, when $k_0 \to 0$, the multiplicative form does <u>not</u> become untenable; in fact, in the limit it approaches the additive form (Keeney and Raiffa[1976]). Thus, we attempt to solve the problem without constraint (23), and if k_0 becomes sufficiently close to zero we switch to the additive form.

4.3 <u>Solution of the Parameter-Estimation Problems</u>

By virtue of Lemmas 1 and 2, we can treat the parameter estimation problems as nonlinear programs with simple bounds. The method we have chosen for solving these problems is the BFGS (Broyden-Fletcher-Goldfarb-Shanno) quasi-Newton algorithm. This is usually described as an unconstrained nonlinear programming technique (c.f., Dennis and More[1977], Fletcher[1980] or Luenberger[1984]), but it can be adapted for bound constraints quite effectively. One modification is to constrain the line search so that bounds are not violated. As for direction finding, let H_k denote the approximated inverse Hessian matrix at the k^{th} iteration of BFGS. (Hence, the search direction is $-H_k g_k$, where g_k is the current gradient.) Fletcher[1980] demonstrates that if the j^{th} row and column of H_k is zeroed, then H_{k+1} will retain these zeroes and the j^{th} variable will not change in the $k+1^{st}$ iteration. So if a variable moves to its bound during a BFGS update, we zero out its row and column in H. In subsequent iterations, we check the corresponding partial derivative, and if the sign is appropriate for leaving the bound in the feasible direction we insert a ± 1 in the corresponding diagonal element of H. Our implementation of this modified BFGS uses the Goldstein-Armijo conditions (c.f., Fletcher[1980]) for sufficient improvement

per iteration. It also checks each proposed search direction to verify that it is a descent direction. Infrequently, it is an ascent direction due to roundoff error. In that case the program reverts to steepest descent by resetting H to identity, and then continues again with the modified BFGS algorithm.

If the sum-of-squares objective function, (13) or (18), of the parameter-estimation problem were strictly quasiconvex, then we would be guaranteed that the point to which the modified BFGS converges is a global optimum. Unfortunately, these functions do not necessarily satisfy strict quasiconvexity. (We have found counterexamples.) However, we conducted a large amount of empirical testing that indicates local optima are unlikely. For each example in a set of parameter estimation problems, we restarted the modified BFGS at a number of distinct starting points. After transforming the result according to Lemma 1 or 2, we found that the modified BFGS converged to the same point regardless of the starting point <u>every</u> time. In another set of tests on these problems, the inequality that defines strict quasiconvexity (c.f., Bazarra and Shetty[1979]) was tested at millions of points over a grid of the feasible region and turned out to be satisfied more than 98% of the time. Considering that our purpose in solving the nonlinear program is to construct an <u>approximate</u> value function for <u>one</u> iteration of an <u>interactive</u> algorithm (for a <u>nonrigorously</u> defined problem), the small risk of local optima is bearable. The alternative of constructing some type of nonconvex optimization scheme is probably not computationally feasible and is certainly not cost-justified.

5. Recovery from Unsuccessful Moves

If our approximating explicit value function is sufficiently poor, then optimizing it may actually lead to a less preferred solution than the one we started from. Oppenheimer[1978] remarked that this problem may arise because value functions are typically much more accurate in the small (i.e., in the vicinity of the point where they were assessed) than in the large. We recover from this situation by reverting to a purely implicit iteration. In this fashion, the implicit/explicit algorithm maintains the global convergence property of the implicit approach, while substantially improving the local convergence.

Our approach can tolerate some degree of inconsistency in the decision maker's responses, especially early in the interactive process. We have observed that decision makers are most likely to exhibit inconsistent behavior early in the interactive process, as they "learn" the effects of tradeoffs. Our implicit/explicit approach deals with this behavior by limiting m, the number of MRS observations to be used in the parameter estimation. Our estimation of the underlying value function needs enough information to make it reasonably accurate, yet not so much information so that inconsistent choices are "remembered" indefinitely. The value $m=5$ accomplishes this in our forestry example.

6. Summary of the Implicit/Explicit Approach

We summarize the results of the preceding sections into the following statement of an approach for multiobjective optimization.

Step 0: [Initialization] Choose $\mathbf{x}^1 \in X$, and τ a convergence tolerance, set k=1, j=1, and m=the number of observations used in approximating the decision maker's explicit utility function. We require $m \geq K$.)

Step 1: [MRS Assessment] Assess the decision maker's tradeoffs at the current point, $\mathbf{x}^k$. (We use a modified form of Dyer's[1973] procedure.) This results in $y_2^k, \ldots, y_K^k$. If j<m go to Step 4, otherwise set j=0 and go to step 2.

Step 2: [Derivation of the Explicit Value Function] Solve a parameter estimation nonlinear programming problem, (13-16) or (18-23), to derive an explicit representation of the decision maker's value function, based on the past m observations. If the assumed form of the value function is additive use the modified BFGS algorithm of Section 4.3 and then apply Lemma 1. If the assumed form is multiplicative, use the modified BFGS and Lemma 2.

Step 3: [Maximization of the Explicit Value Function] Solve to optimality the explicit value maximization problem, (1), resulting in the solution $\mathbf{z}^k$. If $\mathbf{f}(\mathbf{z}^k)$ is preferred to $\mathbf{f}(\mathbf{x}^k)$ go to Step 5, otherwise go to Step 4.

Step 4: [Direction-Finding Subproblem for the Implicit Value Function]

$$\text{Solve} \quad \text{maximize} \sum_{i=2}^{K} y_i^k \, \nabla_{\mathbf{x}} f_i(\mathbf{x}^k) \cdot \mathbf{z}, \text{ s.t. } \mathbf{z} \in X,$$

resulting in $\mathbf{z}^k$.

Step 5: [Step-Size Limit] Set $\mathbf{d}^k = \mathbf{z}^k - \mathbf{x}^k$. If Step 4 was executed set $\bar{t}=1$, otherwise determine the step-size maximum $\bar{t}=\max\{t \mid \mathbf{x}^k + t\mathbf{d}^k \in X\}$.

Step 6: [Line Search] Determine the step size parameter, t^k. That is, find the most preferred solution on the line $\{\mathbf{x}^k+t\mathbf{d}^k | 0 \le t \le \bar{t}\}$. (We use Dyer's[1973] interactive line-search method which requires only paired comparisons of elements in objective space.) Set $\mathbf{x}^{k+1}=\mathbf{x}^k+t^k\mathbf{d}^k$, $k=k+1$, $j=j+1$. If $|\mathbf{x}^k-\mathbf{x}^{k-1}| < \tau$, stop. Otherwise go to Step 1.

To conclude the methodological section of this paper, we offer a few comments. First, note that the features that distinguish the implicit/explicit method from the purely explicit approach are Steps 2 and 3, and they both require solution of a nonlinear programming problem. In order for the method to be useful in practice, it is imperative that these programs be solved very quickly. This is because the method also calls for interaction (in Steps 1 and 6), and most decision makers are unwilling to tolerate long response times during the dialog. The modified BFGS used in Step 2 has been adequate in this regard. As for the Step 3 nonlinear program, we must address this issue on an application-specific basis, and do so in the next section.

Second, as intended, the implicit/explicit method has the implicit method's advantage of a light information load on the decision maker. He or she only has to make binary comparisons of points in objective space, V. The dimension of V, which is equal to the number of objective functions, is typically small (less than eight), so that making these comparisons is not difficult.

Third, the implicit/explicit method inherits another advantage of the implicit approach, namely, it permits the decision maker to learn during the person-computer interaction. Often a decision maker gains new insight into the dynamics of a problem through the solution procedure. Therefore the

decision maker's preference structure may change throughout the course of an interactive dialog. Research by Dickmeyer[1983] reinforces this benefit of person-computer interaction.

Finally, the implicit/explicit approach attempts to exploit the advantages of explicit methods. To hasten convergence it periodically formulates an approximate explicit representation of the multiobjective problem derived from the previous responses of the decision maker, and then solves this problem to optimality without requiring human intervention.

7. Forestry Application

The remaining outstanding question from Section 2 is: how do we solve the explicit value maximization problem? As noted earlier, this issue requires an application-specific approach. Here we describe the forestry application that motivated our work.

Forest management involves the following decisions (Smith[1962]):

1) when to harvest timber,
2) how intensively to harvest,
3) how to regenerate the forest after harvest (natural regeneration versus planting),
4) the nature and timing of cultural treatments such as fertilization and pruning.

A set of specifications of these decisions is called a harvesting regime (or simply a regime). The first two of the above decisions are particularly critical because they must be made far in advance of the others. Since our model is used for long-range planning, we are primarily concerned with these two early decisions, the timing and intensity of harvests.

Forest managers seldom apply the same regime to an entire landholding. Rather, they partition the property into <u>stands</u> and specify regimes for each stand individually. Delineating stand boundaries is often difficult, and is part of the forester's art. Care must be exercised because the partitioning of the forest into stands can have a major influence on the outcome of the planning and analysis. The guiding principle is to obtain approximate homogeneity within each stand of speciation, age, site index (a measure of forest productivity), and stand density.

The goal of forest management is to maximize the "utility" of the forest property to the landowner through time. In the past this use of the term utility has been construed as maximizing volume growth or maximizing economic return. More recently it has been interpreted to include multiple management objectives. An important enunciation of this concept was the Multiple Use and Sustained Yield (MUSY) Act of 1960. MUSY is one of the statutes that define management practices on the U.S. National Forests. The act provides for

> ... the management of all the various surface resources of the National Forests so that they are utilized in the combination that best meets the needs of the American people ... and not necessarily the combination of uses that will give the greatest dollar return ... (MUSY Act of 1960 Section 4(a))

The literature on optimization applications in forest management planning is extensive and diverse. (See Bare et al.[1984] for a review.) Before describing the formulation of our forest management model, it is necessary to provide some background concerning the target user group. It turns out that the special needs of this group have rarely been addressed in this literature, in spite of the group's controlling interest in the U.S. timber supply.

7.1 The NIPF Sector of the U.S. Forest Economy

There are three kinds of owners of forest lands in the United States: 1) government, 2) corporations, and 3) nonindustrial private forest landowners (NIPFs). The NIPFs are individuals who own small tracts (typically less than 100 acres) and who generally do not engage in forestry as a prime source of income.

NIPFs own the majority (58%) of forest land in the United States (USDA Forest Service[1978b]), and thus have an important influence on the nation's potential supply of timber. The influence is particularly strong in the Southeastern states where NIPFs own 71% of the commercial forest land (Ross[1980]).

Lands in the NIPF sector have historically yielded less timber per acre than industrial forest lands. Some factors contributing to this productivity difference are beyond the immediate control of the NIPF, such as differences in land quality and economies of scale in the industrial sector. But it has also been documented in numerous studies (reviewed by Kurtz and Crouse[1981]) that NIPFs underutilize forest management practices and information. The studies have shown that increasing the use of intensive management could lead to greater yields, which would benefit both the NIPF and the nation as a whole. According to Satterfield[1980], the NIPF's deficiency in forest management is due to:

1) the long period between investment in forest management and the realization of a return at harvest,
2) a lack of capital to make investments,
3) a lack of information as to what an individual's land can produce, and how to manage to achieve that potential,

4) perceived low rates-of-return on investments,

5) diverse objectives of land ownership beyond just financial return.

7.2 Lessons From Past Experience

Providing the NIPF with a detailed forest management plan is a time-consuming activity. It first requires considerable effort in the field to measure the landowner's resources. It then requires assessment of the landowner's objectives and the generation of alternative management strategies based on these objectives. The Tennessee Valley Authority developed a computer-assisted tool for NIPF management planning. This tool, known as the Woodland Resource Analysis Program (WRAP) (Hamner[1975], Harrison and Rosenthal[1986]), recommended harvesting strategies using an explicit multiobjective optimization approach. A nonlinear additive explicit value function was assessed with a simple questionnaire administered in the field and it was maximized by a coordinate ascent search. During the period 1977-1983, the program was used by more than 1800 landowners, representing hundreds of thousands of acres of NIPF land.

The program was extremely well-received in the NIPF sector. In a study by the USDA Forest Service[1978a] of NIPFs who had used WRAP, 90 percent found the program sufficiently understandable, 76 percent found it sufficiently personalized, and 97 percent believed that WRAP was applicable to their situation.

However, in spite of this reception, there were three major areas of deficiencies in WRAP.

1) WRAP's multiobjective optimization methodology had to be very simple because of the solution environment that was imposed upon its

design. The assessment of the landowner's preferences was confined to a single brief interview, conducted by a forester, without the benefit of computers. The forester's data were submitted for batch processing on a mainframe computer, and the results of the optimization were presented to the landowner about two weeks later. (WRAP's came into use in 1977, prior to the widespread availability of microcomputers.)

2) WRAP optimized each stand individually. Thus it did not permit either the use of constraints dealing with the entire landholding or the ability to tradeoff sacrifices of benefits on one stand for increases in another.

3) The simulation models that were used to predict the response of the forest to various harvesting regimes needed improvement. They were not uniformly providing face-valid results in the view of experienced foresters.

Based on our experience with 1800 users, and based on our perceptions of both the positive and negative aspects of WRAP, we designed and implemented a new system. It is called FORMAN or Forest Management Planning System. FORMAN's design goals included:

1) the ability to directly address multiple, conflicting objectives,

2) the requirement of no more than one session with the landowner to elicit preference information,

3) a microcomputer implementation,

4) an interactive approach,

5) an integrated management of the entire forest property, not just one stand at a time,

6) the ability to impose property-wide constraints on resources, such as yearly requirements of cashflow and labor, and

7) improved models for simulating forest growth and yield, and for simulating wildlife benefits.

7.3 Formulation of the FORMAN Model

INDICES:

i - stands

j - harvesting regimes

k - objective functions

t - time periods (years).

The ranges of these indices, throughout our discussion, are $i=1,\ldots,I$, $j=1,\ldots,J_i$, $k=1,\ldots,K$, and $t=1,\ldots,T$, where I, J_i, K, and T are given data.

PRIMARY DECISION VARIABLES:

x_{ij} = the percentage of stand i to be managed under regime j, with $0 \le x_{ij} \le 1$.

These variables can take on fractional values, which are interpreted as recommendations to subdivide stands. For example, if $x_{ij} = 1/2$, then stand i will be partitioned into two or more substands, one of which contains half the acreage and is managed by regime j. As will be seen, there is a constraint on the amount of subdivision which can take place. For this purpose we define:

AUXILIARY DECISION VARIABLES:

$\delta_{ij} = 1$ if $x_{ij} > 0$, 0 otherwise.

OBJECTIVE FUNCTIONS:

f_1 - the amount of wood harvested,

f_2 - the expected present value from the sale of timber products,

f_3 - a measure of habitat suitability for deer,

f_4 - a measure of habitat suitability for squirrel, and

f_5 - a measure of habitat suitability for quail.

Obviously there are many other forest benefits of interest to forest landowners; but we wished to limit the objectives to a manageable number, especially in an interactive procedure. The group above is based on past experience with assessing landowner's concerns in the NIPF sector. Moreover, these are benefits for which quantitative relationships exist that permit their inclusion in an optimization process.

Some comments on the wildlife benefits are in order. FORMAN's use of habitat suitability as a model of wildlife benefit is in contrast to WRAP, which attempted to use actual population levels. One reason for this design change is that habitat suitability is much more accurately determined than population. Another reason for the change is it reflects the fact that timber harvesting decisions have only indirect effects on animal populations whereas they have direct and measurable effects on habitat suitability. Our habitat suitability measures range from zero, which is totally unsuitable habitat, to one, which is ideal habitat. See Harrison[1983] and Williamson[1983] for a detailed technical discussion on the derivation of these benefit models.

The three wildlife species who habitats are modeled in FORMAN were chosen because they are associated with the various stages of forest succession. High quail production most often occurs in open fields or young stands. Deer habitat requirements are well satisfied by medium-age stands (of the proper

type). And squirrel habitat is best met by mature, mast-producing stands. Therefore these wildlife species tend to be conflicting in their habitat requirements, which has useful theoretical implications for multiobjective optimization. Keeney and Raiffa[1976] point out the importance of removing correlations between objectives to avoid the problem of redundancy in the analysis of impacts.

The objective functions are linear,

$$f_k(\mathbf{x}) = \sum_i \sum_j \sum_t c_{ijkt}\, x_{ij} \qquad \text{for } k = 1, \ldots, 5 \tag{24}$$

where the coefficients are

c_{ij1t} = volume of timber removed (cubic feet) in year t on tract i under harvesting regime j,

c_{ij2t} = present worth in dollars of management activities in year t on tract i under harvesting regime j,

c_{ij3t} = acreage-weighted habitat suitability index for deer in year t on tract i under regime j,

c_{ij4t} = acreage-weighted habitat suitability index for squirrel in year t on tract i under regime j,

c_{ij5t} = acreage-weighted habitat suitability index for quail in year t on tract i under regime j.

These coefficients (and their associated regimes) are generated by a simulation program that is executed prior to invoking the multiobjective optimization program (Hepp and Williamson[1985]).

There are three classes of constraints in the FORMAN model.

STAND USE CONSTRAINTS:

$$\sum_j x_{ij} \le 1, \quad \forall\, i \tag{25}$$

These generalized upper bound constraints prevent us from prescribing harvesting regimes to more acreage than is available.

CASHFLOW AND WOODFLOW CONSTRAINTS:

$$b_{kt} \leq \sum_i \sum_j c_{ijkt}\, x_{ij} \leq u_{kt} \qquad k=1,2,\text{ and } \forall\ t \tag{26}$$

where b_{kt} and u_{kt} are user-specified lower and upper bounds on cashflow (k=1) and woodflow (k=2) in year t. These are linking constraints in the sense that they could not have been handled had the model been designed (like WRAP) to treat each stand separately.

In practice, there are a variety of reasons for including these constraints. For example, a landowner might wish to limit the cashflow in any one year because of the corresponding tax liability, preferring to spread this return over a few years. Alternatively, a landowner might wish to ensure a minimum level of cash in specified future years to meet an anticipated need, such as financing a college education. The timber-flow constraints are likewise grounded in reality. For example, we have worked with landowners who employ timber harvesting crews on a year-round basis. In this case the landowner wishes that the optimization model recommends harvesting schedules that keep his harvesting crew reasonably employed throughout the horizon. Many other examples exist that recommend the inclusion of these linking constraints. It is possible to model other common resources, in addition to cash and wood, with constraints of this form.

PARTITIONED CARDINALITY CONSTRAINTS:

$$\sum_j \delta_{ij} \leq p_i \qquad \forall\ i \tag{26}$$

$$x_{ij} \leq \delta_{ij} \qquad \forall\ i,j \tag{27}$$

$$\delta_{ij} \in \{0,1\} \qquad \forall\ i,j \tag{28}$$

Here p_i is a user-specified positive integer denoting the maximum number of substands into which stand i may be subdivided.

Partitioned cardinality constraints are a generalization of cardinality constraints, as discussed by Tanahashi and Luenberger[1972] and Rubin[1975]. A technique for handling them heuristically is given in the next section.

There are several practical reasons for imposing these constraints. If the landholding is subdivided into too many parts, then implementing a recommended harvest plan may become managerially infeasible. Furthermore, if a substand is too small, then it may never be economical to bring in a harvesting crew. (More generally, the linear objective functions f_k assume that the benefits obtained in a substand are proportionate shares of the c_{ijkt}. But this may be false for very small substands due to the fixed costs of harvesting and management.)

To summarize this section, the FORMAN model of the forest-management problem (FMP) is

$$(\text{FMP}) \quad \max\ U[\mathbf{f}(\mathbf{x})]$$

s.t.

$$b_{kt} \le \sum_i \sum_j c_{ijkt}\, x_{ij} \le u_{kt} \qquad k=1,2 \text{ and } t=1,\dots,T$$

$$\sum_j x_{ij} \le 1 \qquad i=1,\dots,I$$

$$\sum_j \delta_{ij} \le p_i \qquad i=1,\dots,I$$

$$0 \le x_{ij} \le 1 \qquad i=1,\dots,I,\quad j=1,\dots,J_i$$

$$x_{ij} \le \delta_{ij} \qquad i=1,\dots,I,\quad j=1,\dots,J_i$$

$$\delta_{ij} \in \{0,1\} \qquad i=1,\dots,I,\quad j=1,\dots,J_i.$$

where $\mathbf{f}(\mathbf{x})$ is given by (24) and $U[\mathbf{f}(\mathbf{x})]$ is the (unknown) value function of the decision maker.

In practice, the number of stands (I) has ranged from 2 to 10, the number of regimes per stand (J_i) from 5 to 40 and the number of years (T) is usually 30.

7.4 Computer Implementation and Solution

The FORMAN system is an implementation of the implicit/explicit multiobjective optimization approach to the FMP model. Here we focus on two issues of implementation and provide a description of their resolution in FORMAN. Specifically, we deal with:

1) a heuristic solution of partitioned cardinality constrained linear programming (PCCLP) subproblems, and
2) solution of the explicit value maximization problem.

The PCCLP must be solved repeatedly as a subproblem within our algorithm for handling the explicit phase of the implicit/explicit approach. Unfortunately, the PCCLP is NP-complete; because as Wood[1986] has shown, if there were a polynomial-time algorithm for PCCLP, there would also be a polynomial-time algorithm for the "exact cover by 3-sets" problem, which is NP-complete (Garey and Johnson[1979]). Intractability aside, a heuristic approach to the PCCLP is reasonable in the forestry application because:

1) the multiobjective procedure requires the solution of a number of partitioned cardinality constrained LP's in an interactive setting,
2) we implemented FORMAN on microcomputers, and
3) the partitioned cardinality constraints are among the "softest" problem constraints. (Following the approach of Brown and

Graves[1975] and Liebman et al. [1986], we allow some of the constraints to be "elastic" or "soft", which means they may be violated at a linear cost.)

In light of these considerations, we have derived a heuristic technique to deal with the partitioned cardinality constraints. It is based on the following result.

Lemma 3 (Harrison[1983], pp. 123-124)

Given the problem:

$$\begin{aligned} &\max \ \mathbf{cx} \\ &\text{s.t.} \quad \mathbf{Ax} = \mathbf{b} && \text{(PCCLP)} \\ &\qquad | \mathbf{x}_i^+ | \le p_i && i = 1, \ldots, I \\ &\qquad \mathbf{x} \ge \mathbf{0}, \end{aligned}$$

where the variables are partitioned into I mutually exclusive and collectively exhaustive sets, and $| \mathbf{x}_i^+ |$ denotes the number of positive variables contained in the i'th set. If there exists an optimal solution to PCCLP, then there exists a basic optimal solution.

This lemma demonstrates that an extreme point enumeration technique for the PCCLP will eventually find an optimal solution. Returning now to our PCCLP heuristic, it is composed of two rules: 1) when considering variables to price out, we price out first those that will not lead to a violation of cardinality (if any), and 2) we rescale the reduced cost of each variable as it is priced out to reward/penalize a variable with respect to its current contribution to the cardinality constraints. However, the sign of the reduced cost is not changed so as to maintain the guaranteed convergence characteristics of the simplex implementation. The essence of this idea is to

increase the attractiveness of nonbasic variables that will not lead to cardinality constraint violation.

This heuristic does not guarantee satisfaction of the partitioned cardinality constraints, but if they are violated, the violations tend to be dispersed throughout the stands, which keeps the solution managerially feasible.

The final aspect of implementation is the maximization of the explicit value function, as required in Step 3 of our implicit/explicit approach to multiobjective optimization. This problem is given as the forest management problem (FMP), where $U[\mathbf{f}(\mathbf{x})]$ is defined by either (6) or (7). Note that when the partitioned cardinality constraints are handled implicitly by our heuristic procedure, the resulting problem is the maximization of a nonlinear objective function subject to a linear set of constraints. To solve this problem we use the well-known Frank-Wolfe algorithm (Wolfe[1970]).

This algorithm was chosen because of its good initial convergence characteristics (Hogan[1971], Wolfe[1970]), and because it was compatible with existing software in FORMAN. Given that a computer procedure already exists for solving the Frank-Wolfe steps in the Geoffrion-Dyer-Feinberg algorithm, it is relatively easy to implement this approach.

8. Summary and Conclusions

We presented a method for multiobjective optimization that combines the complementary strengths of two existing approaches — implicit utility/value maximization and explicit utility/value maximization. The idea of the combined approach is to embed within the implicit method a procedure which periodically formulates an approximate explicit representation of the

multiobjective problem, and then optimally solves it without user interaction. Operationally, the use of this idea requires fast and frequent solution of two nonlinear programs. The first nonlinear program is for estimating the parameters of the approximate explicit value function. The second nonlinear program is to maximize the explicit value function subject to the constraints of the original problem.

Our development of this approach was motivated by a problem in forest management. This problem concerns the timing and intensity of timber harvests on nonindustrial private forest (NIPF) lands. NIPF lands constitute the majority of commercial timberlands in the U.S., but the operations research/decision sciences literature has been essentially devoid of approaches for meeting the special needs of this important group.

We have developed a model for this forest management problem and an implementation of the implicit/explicit multiobjective approach for solving it. The model and solver are embodied in a system called FORMAN, which is currently undergoing field testing throughout the southeast by a number of academic, government, and private foresters. The Tennessee Valley Authority has provided FORMAN as a replacement for an earlier system (WRAP), which has been used by over 1,800 landowners.

An important aspect of FORMAN's development is that it is implemented on a microcomputer. In this environment it was especially challenging to solve the nonlinear programs quickly enough to be useful within an interactive procedure. The algorithms we employed have met this requirement. (We used a modified BFGS algorithm for the parameter estimation problem and the Frank-Wolfe method with heuristic treatment of the partitioned cardinality constraints for the explicit value maximization problem.)

As a tangential development, the partitioned cardinality constraints are an interesting modeling device which may have applications in other areas. For example, they may apply to a production/blending problem in which x_{ij} is the proportion of product i to be composed of ingredient j. One may wish to limit the number of ingredients in the product for the sake of efficiencies in purchasing, inventory control and/or processing.

We anticipate that FORMAN will see extensive use in the field as its predecessor did, and we expect that this experience will lead to additional applications of the implicit/explicit approach.

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REFERENCES

Baumol, W. J. 1977. ***Economic Theory and Operations Analysis*** (4th ed.). Prentice-Hall, Englewood Cliffs, NJ.

Bare, B. B., D. G. Brigg, J. P. Roise, and G. F. Schreuder. 1984. A Survey of Systems Analysis Models in Forestry and the Forest Products Industries. ***European Journal of Operational Research*** 18:1-18.

Bazarra, M. S. and C. M. Shetty. 1979. ***Nonlinear Programming Theory and Algorithms***. John Wiley and Sons, New York, NY.

Brown, G. G. and G. W. Graves. 1975. Design and Implementation of a (Mixed Integer, Nonlinear) Optimization System. Presented at the TIMS/ORSA Joint National Meeting, Las Vegas, Nevada.

Chankong, V. and Y. Y. Haimes. 1983. ***Multiple Objective Decision Making: Theory and Methodology***. North Holland, Inc.

Cochrane, J. L. and M. Zeleny. 1973. ***Multiple Criteria Decision Making***. University of South Carolina Press, Columbia, SC.

Cohon, J. L. 1978. ***Multiobjective Programming and Planning***. Academic Press.

Cohon, J. L. and D. M. Marks. 1975. A Review and Evaluation of Multiobjective Programming Techniques. ***Water Resources Research*** 11:208-220.

Dennis, J. E. and J. J. More. 1977. Quasi-Newton Methods, Motivation and Theory. ***SIAM Review*** 19:46-79.

Dickmeyer, N. 1983. Measuring Effects of a University Planning Decision Aid. ***Management Science*** 29:673-685.

Dyer, J. S. 1973. An Empirical Investigation of a Man-Machine Interactive Approach to the Solution of the Multiple Criteria Problem. In Cochrane and Zeleny (Eds.), ***Multiple Criteria Decision Making***, 1973.

Dyer, J. S. and R. K. Sarin. 1979. Measurable Multiattribute Value Functions. ***Operations Research*** 27:810-822.

Evans, G. W. 1984. An Overview of Techniques for Solving Multiobjective Mathematical Programs. ***Management Science*** 30:1268-1282.

Farquhar, P. H. 1984. Utility Assessment Methods. ***Management Science*** 30:1283-1300.

Fishburn, P. C. 1983. ***The Foundations of Expected Utility***. Reidel, Dorecht, The Netherlands.

Fletcher, R. 1980. ***Practical Methods of Optimization***. John Wiley and Sons.

Garey, M. R. and D. S. Johnson. 1979. ***COMPUTERS AND INTRACTABILITY A Guide to the Theory of NP-Completeness***. W. H. Freeman and Co., San Francisco.

Geoffrion, A. M., Dyer, J. S., and A. Feinberg. 1972. An Interactive Approach for Multicriterion Optimzation with an Application to the Operation of an Academic Department. ***Management Science*** 19:357-368.

Goicoechea, A. D. Hansen and L. Duckstein. 1982. ***Multiobjective Decision Analysis with Engineering and Business Applications***. John Wiley and Sons, New York, NY.

Golabi, K., C. W. Kirkwood and A. Sicherman. 1981. Selecting a Portfolio of Solar Energy Projects Using Multiattribute Preference Theory. ***Management Science*** 27:174-189.

Gros, J. 1975. Power Plant Siting: A Paretian Environmental Approach. ***Nuclear Engineering Design*** 34:281-292.

Haimes, Y., Hall, D., and S. Freedman. 1975. ***Multiobjective Optimization on Water Resources Systems***. Elsevier, Amsterdam, The Netherlands.

Hair, D. 1980. Timber Situation in the United States - 1952-2030. ***Journal of Forestry*** 78:683-686.

Hamner, L. N. 1975. WRAP - A Multiple Resource Model for Allocation of Wildland Resources. In ***Systems Analysis and Forest Resource Management***, pp. 127-131. Society of American Foresters, Bethesda, MD.

Hannan, E. L., Smith, J. A. and G. R. Gilbert. 1983. A Multiattribute Decision Making Approach to the Selection of an Auxiliary Device for Icebreakers. ***Decision Sciences*** 14:240-252.

Harrison, T. P. 1983. Design and Development of a Decision Support System for Forest Management Planning with Multiple Objectives. Ph.D. Dissertation, The University of Tennessee. Available through University Microfilms, Ann Arbor, Michigan. ***Dissertation Abstracts International*** 44(10-B), 3177.

Harrison, T. P. and R. E. Rosenthal. 1984. On the Use of Utility/Value Functions in a Multiobjective Mathematical Programming Framework. Working Paper 84-6, Management Science Department, The Pennsylvania State University, University Park, PA.

Harrison, T. P. and R. E. Rosenthal. 1986. A Multiobjective Optimization Program for Scheduling Timber Harvests on Nonindustrial Private Forest Lands. ***TIMS Special Study on Systems Analysis in Forestry and Forestry Industries***, TIMS Studies in Management Science, Volume 21, pp. 269-283.

Hepp, T. E. and J. F. Williamson. 1985. An Overview of TIMPRO: Timber Yield, Investment Analysis, and Wildlife Projection Tool. Presented at Software Solutions - Computer Symposium and Software Fair, April 21-24, 1985, Louisville, KY. Cosponsored by the Forest Resource Systems Institute and the Forest Products Research Society.

Ho, J. K. 1979. Holistic Preference Evaluation in Multiple Criteria Optimization. Working Paper, Management Science Department, The University of Tennessee, Knoxville, TN.

Hobbs, B. F. 1979. Analytical Multiobjective Decision Methods for Power Plant Siting: A Review of Theory and Applications. Report BNL-NUREG-51204. Brookhaven National Laboratory, Upton, New York.

Hogan, W. W. 1971. Convergence Results for Some Extensions of the Frank-Wolfe Method. Working Paper No. 169, Western Management Science Institute, University of California, Los Angeles.

Hwang, C. L. and A.S.M. Masud. 1979. ***Multiple Objective Decision Making — Methods and Applications***. Springer-Verlag.

Keefer, D. L. 1978. Applying Multiobjective Decision Analysis to Resource Allocation Planning Problems. In Multiple Criteria Problem Solving. Proceedings, pp:299-320, S. Zionts ed., Buffalo, New York, USA, Springer-Verlag.

Keeney, R. L. 1977. The Art of Assessing Multiattribute Utility Functions. ***Organizational Behavior and Human Performance*** 19:267-310.

Keeney, R. L. 1979. Evaluation of Proposed Storage Sites. ***Operations Research*** 27:48-64.

Keeney, R. L. 1980. ***Siting Energy Facilities***. Academic Press, New York, NY.

Keeney, R. L. and H. Raiffa. 1976. ***Decisions with Multiple Objectives: Preferences and Value Tradeoffs***. John Wiley and Sons, New York, NY.

Kirkwood, C. W. and R. K. Sarin. 1980. Preference Conditions for Multiattribute Value Functions. ***Operations Research*** 28:225-232.

Kurtz, W. B. and C. K. Crouse. 1981. Nonindustrial Private Forest Ownership Studies: A Bibliography. Council of Planning Librarians Bibliographies, 1313 East 60th Street, Chicago, Illinois.

Liebman, J., Lasdon, L., Schrage, L. and A. Waren. 1986. ***Modeling and Optimization with GINO***. The Scientific Press.

Luenberger, D. G. 1984. ***Introduction to Linear and Nonlinear Programming***. Addison-Wesley.

Multiple Use Sustained Yield Act. 1960. 74 STAT. 215.

Oppenheimer, K. R. 1978. A Proxy Approach to Multi-attribute Decision Making. ***Management Science*** 24:675-689.

Ringuest, J. L. and T. R. Gulledge. 1983. A Preemptive Value Function Method Approach for Multiobjective Linear Programming Problems. ***Decision Sciences*** 14:76-86.

Rosenthal, R. E. 1985. Principles of Multiobjective Optimization. ***Decision Sciences*** 16:133-152.

Ross, E. W. 1980. Nonindustrial Forest-Land Owners in the Southeast. in ***Proceedings of the Eighth Annual Hardwood Symposium***, Hardwood Research Council, Asheville, North Carolina.

Roy, B. and P. Vinke. 1981. Multiattribute Analysis: Survey and New Directions. ***European Journal of Operations Research*** 8:207-218.

Rubin, D. S. 1975. Vertex Generation and Cardinality Constrained Linear Programs. ***Operations Research*** 23:555-565.

Satterfield, S. 1980. A Perspective From the Office of Management and Budget. ***Nonindustrial Private Forests: Data and Information Needs Conference Proceedings.*** J. P. Royer and F. J. Convery, Editors. Duke University, Durham, North Carolina, pp. 11-14.

Sedjo, R. A. and D. M. Ostermeier. 1977. Policy Alternatives for Nonindustrial Private Forests. Report of the Workshop on Policy Alternatives for Nonindustrial Private Forests, Sponsored by the Society of American Foresters and Resources for the Future.

Smith, D. M. 1962. ***The Practice of Silviculture***. John Wiley and Sons.

Tanahashi, T. and D. G. Luenberger. 1972. Cardinality-Constrained Linear Programming. Report SE-79R-5, Department of Engineering-Economic Systems, Stanford University.

USDA Forest Service. 1978a. Evaluation of the Alabama Pilot Test of the Woodland Resource Analysis Program (WRAP). Nathan Byrd, USFS, SA - State and Private Forestry, Chairman.

USDA Forest Service. 1978b. Forest Statistics of the U. S., 1977.

Williamson, J. F. 1983. Wildlife Benefit Simulations in the FORMAN Program. Technical Note B48, Tennessee Valley Authority, Office of Natural Resources, Norris, TN 37828.

Wolfe, P. 1970. Convergence Theory in Nonlinear Programming. in J. Abadie (ed.), ***Integer and Nonlinear Programming***, North-Holland.

Wood, R. K. 1986. personal communication.

Zeleny, M. 1982. ***Multiple Criterion Decision Making***. Prentice-Hall.

Zionts, S. and J. Wallenius. 1976. An Interactive Programming method for Solving the Multiple Criteria Problem. ***Management Science*** 22:652-663.

Zionts, S. and J. Wallenius. 1983. An Interactive Multiple Objective Linear Programming Method for a Class of Underlying Nonlinear Utility Functions. ***Management Science*** 29:519-529.

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